Upper-layer Survivability in Layered Networks with Lower-layer Capacity

Zhili Zhou
Singapore Research Collaboratory
IBM, Singapore, 486072

Tachun Lin
Department of Computing and Technology
Cameron University, Lawton, OK 73505

Krishnaiyan Thulasiraman
School of Computer Science
University of Oklahoma, Norman, OK, 73019

Abstract

The upper-layer survivability problem in layered networks is to map each link \((u, v)\) in the upper-layer into a lightpath between nodes \(u\) and \(v\) in the lower-layer network such that failures experienced in the lower-layer network does not cause the upper-layer network to become disconnected. In this paper, we focus on upper-layer network survivability problems to cope with both capacity and survivable issues. We formulate this problem by mixed-integer programs, prove NP-hardness of this problem, and explore the limitation imposed on routings by lower-layer capacity. A heuristic approach and a Benders’ decomposition approach are proposed. Computational experiments using data based on NSFNET and some European networks demonstrate that the proposed modeling and solution approaches are efficient and effective.

Keywords
Layered communication network, survivability, mixed-integer programming, capacity assignment, Benders’ decomposition

1. Introduction

Network survivability is among the most recurring issues when designing telecommunication networks. When a network facility (link or node) fails, a mechanism which guarantees continued network flow and operability is critical. Over the last decade there has been an explosive growth in Internet traffic requiring high transport capacity of telecommunication networks. Most telecommunication networks are layered networks. In this paper, the layered network, which we discuss, is based on architectures of a layered telecommunication network – IP-over-WDM network. IP-over-WDM network is a two-layered telecommunication network where an IP (logical/upper-layer) network is embedded onto a WDM (physical/lower-layer) network. Wavelength Division Multiplexing (WDM) networks where each optical fiber may carry the bandwidth in terabits per second are usually used in backbone networks. IP routers and OXCs correspond to the logical and physical nodes. Links connecting nodes in a logical network are called the logical links, and the physical links are realized via optical fibers. The logical nodes are commonly assumed to have corresponding nodes in the physical network. On the other hand, not all physical nodes may exist in the logical network. A router-to-router link is implemented through a wavelength on a path between two end nodes in a WDM network bypassing opto-electro-optic (O-E-O) conversions on intermediate nodes in the path. This path is called a lightpath. Each optical fiber may carry multiple lightpaths; hence, a failure on an optical fiber may have a cascading effect causing failures on multiple logical links, resulting in a huge amount of data traffic (terabytes/sec) loss. This has given rise to an extensive interest in the study of survivability issues in the IP-over-WDM network, which guarantees the connectivity of all logical links under arbitrary physical link failure by lightpath routing in a given network and is different from the design of survivable network topology.
In practice, physical link capacities and logical link demands are usually considered during design phase to reduce costs, especially for the IP-over-WDM network in the metropolitan area (see [11] and [12]). But most previous research concentrated on survivable routing design for uncapacitated IP-over-WDM networks. These research works could be separated as two main directions, the mixed-integer programming based approach and graph theory based heuristic approach. For the mixed-integer programming based approach, Modiano and Narula-Tam [10] proved a necessary and sufficient condition for survivable routing under a single failure in IP-over-WDM networks and formulated the problem as an Integer Linear Program (ILP). Todimala and Ramamurthy [14] adapted the concept of Shared Risk Link Group introduced in [13] and also computed the routing through an ILP formulation. Extensions of the work in [10] are given in [7] and [5]. [7] introduced certain connectivity metrics for layered networks and provided ILP formulations for the lightpath routing problem satisfying these metrics. In particular, they provided approximation heuristics for lightpath routing maximizing the min cross layer cut metric. This metric captures the robustness of the networks after multiple physical link failures. Kan et al. [5] discussed the relationship between survivable lightpath routing and the spare capacity requirements on the logical links to satisfy the original traffic demands after failures. For the graph theory based heuristics, Kurant and Thiran [6] proposed the Survivable Mapping by Ring Trimming (SMART) framework which first attempts to find link-disjoint mappings for the links of a subgraph of the given logical graph. Another approach proposed by Lee et al. [8] utilized the concept of ear-decomposition on bi-connected topologies. One can show that this is, in fact, a special variant of the framework given in [6], though it was developed independently. Javed et al. obtained improved heuristics based on SMART [3] [4]. Using duality theory in graphs, a generalized theory of logical topology survivability was given by Thulasiraman et al.

There are few research works on the survivability for capacitated IP-over-WDM network. Kan et al. [5] studied logical protection and the impact of lightpath routing on network survivability. They proposed a general strategy to consider the logical spare capacity and diverse physical lightpaths to reserve disrupted logical traffic. But their problem setting is without the consideration of logical demands and physical link capacity. Meanwhile, they decomposed lightpath routing and spare capacity assignment as two subproblems. Lin et al. [9] studied the weakly and strongly survivable routing with physical protection with physical link capacity. They separated the survivable IP-over-WDM with physical capacity as two subproblems, the survivable routing in uncapacitated IP-over-WDM network and capacity assignment based on survivable routing.

We demonstrate the difference between the survivable routing for un-capacitated layered network and capacitated layered network with the following example. Figures 1(a)(b) show a upper-layer network with demands on its links and a lower-layer network with capacities on its links. A survivable routing satisfying both upper-layer link demands and guaranteeing upper-layer network survivability after a single lower-layer link failure is shown in Fig. 1(c). For the mappings in Figs. 1(d)(e), either the upper-layer network survivability criterion will not be satisfied after a lower-layer link failure.

![Figure 1: Capacitated survivability and demand satisfaction](image-url)

We define the upper-layer network survivability in the capacitated layered network as the ability of the network to recover all upper-layer demands under any single physical link failure. In the rest of this paper, we consider upper-
layer network survivability in a layered network with capacities on lower-layer and demands on upper-layer links. Different from [9], we consider the upper-layer network protection. We build the connection of the lower-layer link capacity and upper-layer link spare capacity and adapt the upper-layer protection to achieve survivability with lower-layer capacity limitation. Our model integrates the lightpath routing and spare capacity assignment as a single problem by the necessary and sufficient condition for upper-layer survivability. To our best knowledge, none of this previous research has considered this issue.

The rest of the paper is organized as follows. Section 2 provides formal definitions of survivability for capacitated layered network. Section 3 provide heuristics and Benders’ decomposition based algorithm to solve the problem. We provide the preliminary computational results in Section 4.

2. Problem Description

We use the terms network and topology, edge and link, node and vertex, logical and upper-layer, physical and lower-layer, interchangeably throughout the paper. Let \( G_U = (V_U, E_U) \) be a upper-layer network and \( G_L = (V_L, E_L) \) be a lower-layer network in a two-layer network. Let \((i, j)\) be a lower-layer link and \((s, t)\) be an upper-layer link. Capacity on lower-layer link \((i, j)\) is \( c_{ij} \) and demand on upper-layer link \((s, t)\) is \( d_{st} \). We assume both the upper and lower-layer networks are two-edge connected. We let \( CS(S, V_U - S) \) be a cutset in the upper-layer network.

To indicate the lightpath of upper-layer link, we let \( y_{st}^m \) be a binary variable that indicates whether the logical link \((s, t)\) remains connected after any lower-layer link failure, and there exists sufficient capacity on lower-layer network to support all disrupted traffic.

**Definition 1.** A layered network with upper-layer network, \( G_U = (V_U, E_U) \), and lower-layer network, \( G_L = (V_L, E_L) \), is called upper-layer survivable if after any lower-layer link failure, \( G_U \) remains connected and all demands are satisfied.

**Definition 2.** The upper-layer spare capacity is the extra demand request on upper-layer link \((s, t)\) to satisfy disrupted upper-layer demands after any lower-layer link failure.

**Definition 3.** The lower-layer spare capacity is the extra capacity on lower-layer link \((i, j)\) to satisfy disrupted upper-layer demands after any physical link failure.

We denote \( C_{st} \) as the upper-layer spare capacity on link \((s, t)\) and \( S_{ci} \) as the lower-layer spare capacity on link \((i, j)\).

**Definition 4.** The upper-layer network protection for survivability is to utilize upper-layer spare capacity to provide sufficient capacity for unsatisfied upper-layer demands in the upper-layer network and add lower-layer spare capacity to support upper spared capacity and demand against any lower-layer link failure.

Based on Definition 4 to achieve upper-layer network survivability, the upper spare capacity should provide sufficient capacity to recover disrupted traffic in the upper-layer network. Then, the capacity on lower-layer link would support demands and spare capacities carried by lightpaths which corresponds to upper-layer links. Meanwhile, the connectivity of upper-layer network should be guaranteed after any lower link failure. Now, we explore the necessary and sufficient conditions for upper-layer network protection to achieve survivability. First, we demonstrate the necessary and sufficient condition that the upper-layer spare capacity could recover unsatisfied upper-layer demands with any single lower-layer link failure.

**Proposition 1.** The necessary and sufficient condition for upper-layer spare capacity to recover unsatisfied upper-layer demands against any single lower-layer link failure is \(\sum_{(s, t) \in CS(S, V_U - S)} (y_{st}^m + y_{ji}^d) d_{st} \leq \sum_{(s, t) \in CS(S, V_U - S)} (1 - y_{ij}^f) C_{st}, S \in V_U, (i, j) \in E_L\) (1)

**Proof.** If the lightpath of upper-layer link \((s, t)\) routes through \((i, j)\), \(y_{st}^m + y_{ji}^d = 1\); otherwise, \(y_{st}^m + y_{ji}^d = 0\). Hence, after \((i, j)\) failure, if \((y_{st}^m + y_{ji}^d) = 1\), then, \(d_{st}\) cannot be satisfied. Following the maximum-flow and minimum-cut theorem and satisfying all disrupted upper-layer demands, the total disrupted upper-layer demand in a cutset under a physical link \((i, j)\) failure, i.e., \(\sum_{(s, t) \in CS(S, V_U - S)} (y_{st}^m + y_{ji}^d) d_{st}\) is less than or equal to spare capacity on the remaining upper-layer links in the same cut-set, i.e., \(\sum_{(s, t) \in CS(S, V_U - S)} (1 - y_{ij}^f) C_{st}\). \(\square\)
Second, we present the necessary and sufficient condition that the lower-layer spare capacities could support the upper-layer demands and spare capacities.

**Proposition 2.** Let $C_st$ be sufficient upper-layer spare capacity, then, the necessary and sufficient condition of lower-layer spare capacity to support $C_st$ is

$$Sc_{ij} = \max \left\{ \sum_{l(t) \in E_L} (C_st + d_{st})(y_{ij}^m + y_{ij}^n), c_{ij} \right\} - c_{ij}, \quad (i, j) \in E_L$$  \hspace{1cm} (2)

**Proof.** With upper-layer spare capacity, the lightpath $P_{st}$ for upper-layer link $(s, t)$ carries $C_st + d_{st}$ as upper-layer demand through lower-layer link. If $y_{ij}^m + y_{ij}^n = 1$, then, $(i, j) \in P_{st}$, then, capacity and spare capacity of $(i, j)$ would be at least equal to total demands of upper-layer links whose lightpaths are routed through $(i, j)$, i.e., $c_{ij} + Sc_{ij} \geq \sum_{l(t) \in V_U} d_{st} + C_st$ with $(i, j) \in P_{st}$. If $c_{ij} \geq d_{st} + C_st$, then, $Sc_{ij} = 0$; otherwise, $Sc_{ij} = \sum_{l(t) \in V_U} (d_{st} + C_st) - c_{ij}$. 

Third, Modiano and Narula-Tam in [10] provided the necessary and sufficient condition for upper-layer connectivity after any lower-layer link failure.

**Theorem 1.** The upper-layer network remains connectivity if and only if for every cut-set $CS(S, V_U \setminus S)$ of the upper-layer network the following holds,

$$\sum_{(s,t) \in CS(S, V_U \setminus S)} (y_{ij}^m + y_{ij}^n) \leq |CS(S, V_U \setminus S)| - 1$$  \hspace{1cm} (3)

With Proposition 1 and 2 and Theorem 1, the following conclusion holds.

**Theorem 2.** The necessary and sufficient condition to guarantee upper-layer network survivability is that the upper-layer spare capacity, lower-layer spare capacity, and lightpath routing satisfy (1), (2), and (3).

**Proposition 3.** The problem of guaranteeing survivability through upper-layer protection is NP-complete.

Modiano and Narula-Tam [10] proved that the survivable lightpath routing design to guarantee the upper-layer network connectivity after an arbitrary lower-layer link failure is NP-complete. The upper-layer protection for upper-layer survivability seeks lightpath routing to remain connectivity of the upper-layer network with spare capacity assignment, which is a superset of Modino and Narula-Tam’s problem. Hence, the upper-layer protection for upper-layer survivability is NP-complete.

Now, we propose the mixed-integer programming formulation for the upper-layer survivability with upper-layer protection as follows. The objective is to minimize the total lower-layer spare capacities.

$$(ULS) \min \sum_{y,Sc} \sum_{(i,j) \in E_L} Sc_{ij}$$

s.t. \hspace{1cm} $\sum_{(i,j) \in E_L} y_{ij}^m - \sum_{(j,i) \in E_L} y_{ij}^n = 1$, if $s = i, (s, t) \in E_U$  \hspace{1cm} (4)

$$\sum_{(i,j) \in E_L} y_{ij}^m - \sum_{(j,i) \in E_L} y_{ij}^n = -1$, if $t = i, (s, t) \in E_U$  \hspace{1cm} (5)

$$\sum_{(i,j) \in E_L} y_{ij}^m - \sum_{(j,i) \in E_L} y_{ij}^n = 0$, otherwise, $(s, t) \in E_U$  \hspace{1cm} (6)

$$y_{ij}^m + y_{ij}^n \leq 1$, $(s, t) \in E_U, (i, j) \in E_L$  \hspace{1cm} (7)

$$Sc_{ij} = \max \left\{ \sum_{l(t) \in E_U} (C_st + d_{st})(y_{ij}^m + y_{ij}^n), c_{ij} \right\} - c_{ij}, \quad (i, j) \in E_L$$  \hspace{1cm} (8)

$$\sum_{(s,t) \in CS(S, V_U \setminus S)} (y_{ij}^m + y_{ij}^n) \leq |CS(S, V_U \setminus S)| - 1$$  \hspace{1cm} (9)

$$y_{ij}^m \in \{0, 1\}, \quad (s,t) \in E_U, (i,j) \in E_L$$  \hspace{1cm} (10)

$$C_st \geq 0, Sc_{ij} \geq 0, \quad (s,t) \in E_U, (i,j) \in E_L$$  \hspace{1cm} (11)
Constraints (4), (5), and (6) with binary variable $y_{ij}^0$ provide routings for logical pairs with single unit flow. This is achieved by requiring the binary decision variables $y_{ij}^0$ to satisfy the flow constraints. The physical links for which $y_{ij}^0 = 1$ define a single lightpath for each logical link $(s,t)$. Note that linkpath constraints (4), (5), and (6) do not eliminate the loop in the lightpath. Hence, constraint (7) is introduced to remove loops in lightpaths. The last three constraints guarantee enough upper-layer and lower-layer spare capacities to support survivable upper-layer lightpath routing in the layered network.

### 3. Solution Approach

In this section, we discuss solution approach for the upper-layer protection for upper-layer survivability. First, we apply the Benders’ decomposition approach to generate valid cuts to ULS to solve small size layered network. In order to solve medium to large size layered network, we propose a heuristic.

#### 3.1 Benders’ Cuts

Before introducing Benders’ decomposition framework to generate valid cuts for ULS, we present another necessary and sufficient condition to guarantee the upper-layer network connectivity after an arbitrary lower-layer link failure with upper-layer spanning tree structure and auxiliary variable $r_{ij}^U$. 

\begin{equation}
\sum_{(s,t) \in E_U} r_{st}^U - \sum_{(s,t) \in E_U} r_{ts}^U = -1, \quad \text{if } s = v_1, (i, j) \in E_L \tag{10}
\end{equation}

\begin{equation}
\sum_{(s,t) \in E_U} r_{st}^U - \sum_{(s,t) \in E_U} r_{ts}^U = \frac{1}{|V_U| - 1}, \quad \text{otherwise } (i, j) \in E_L. \tag{11}
\end{equation}

\begin{align}
0 & \leq r_{st}^U \leq 1 - (y_{ij}^0 + y_{ji}^0), \quad (s,t) \in E_U, (i, j) \in E_L \tag{12} \\
0 & \leq r_{ts}^U \leq 1 - (y_{ij}^0 + y_{ji}^0), \quad (s,t) \in E_U, (i, j) \in E_L \tag{13}
\end{align}

**Proposition 4.** The tree protection constraints (10) to (13) provide the necessary and sufficient condition for the upper-layer connectivity in the layered network under lower-layer link failure.

Constraints (10) to (13) guarantee that after an arbitrary lower-layer link failure, a spanning tree exists in the upper-layer network. Hence, the upper-layer network remains connectivity.

Now, we consider the Benders’ decomposition framework to generate cuts. We consider the master problem with the same objective function with ULS and constraints (7) to (6) and slave problem with constraints (10) to (13). The following procedure follows the Benders’ decomposition. We introduce the dual variables according to constraints (10) to (13) as $\alpha_{ij}^U$, $\beta_{ij}^U$ and $\hat{\beta}_{ij}^U$ where $(i, j) \in E_L$, $(s,t) \in E_U$.

With the given $\bar{y}$ solutions which satisfy constraints (4) to (6), we have the following dual formulation of constraint (10) to (13) with objective function $\min \bar{0}$ holds.

\begin{equation}
\max_{\alpha, \beta} \sum_{(i,j) \in E_L, (s,t) \in V_U, s \neq v_1} \left[ |V_U| - 1 \right] \alpha_{ij}^U \alpha_{ij}^j + \sum_{(i,j) \in E_L, (s,t) \in E_U} (-1 + y_{ij}^0 + y_{ji}^0)(\beta_{ij}^U + \hat{\beta}_{ij}^U) \tag{14}
\end{equation}

\begin{align}
\text{(DT) s.t. } \alpha_{ij}^U - \alpha_{ij}^j + \beta_{ij}^U & \leq 0, \quad (i, j) \in E_L, (s, t) \in E_U \tag{15} \\
\alpha_{ij}^U - \alpha_{ij}^j + \hat{\beta}_{ij}^U & \leq 0, \quad (i, j) \in E_L, (s, t) \in E_U \tag{16}
\end{align}

If the given $\bar{y}$ is infeasible for constraints (10) to (13), then, we could generate the feasibility cuts based on the extreme ray of (DT) as follows:

\begin{equation}
\sum_{(i,j) \in E_L, (s,t) \in V_U, s \neq v_1} \left[ |V_U| - 1 \right] \alpha_{ij}^U \alpha_{ij}^j + \sum_{(i,j) \in E_L, (s,t) \in E_U} (-1 + y_{ij}^0 + y_{ji}^0)(\beta_{ij}^U + \hat{\beta}_{ij}^U) \geq 0 \tag{17}
\end{equation}

Hence, the above Benders’ cuts are a new family of constraints for ULS.

There are large families of constraints (1), (2), and (17). We will utilize the branch-and-cut algorithm to add these constraints. Separation algorithms will be generated to add cuts from these constraint families. The following heuristic could provide better initial feasible solution and better upper bound for the branch-and-cut algorithm.

#### 3.2 Heuristic Approach

In this section, we present our heuristics for upper-layer survivability with single lower link failure. We decompose the whole problem into two subproblems and build corresponding heuristics.
Subproblem 1: We design the IP-over-WDM network such that the upper-layer topology remains connected after any lower-layer link failure with the objective of maximizing upper-layer demand satisfaction (upper-layer demands routable under the selected lightpath routing) after any lower-layer link failure.

Subproblem 2: With the information of existing lightpaths and the lower-layer link failure, the demands/flow on the failed lightpaths need to be rerouted and the objective is to minimize the maximum of the unsatisfied demands caused by each lower-layer link failure.

### Algorithm 1 Heuristic for upper-layer connectivity

**Input:** Lower-layer topology $G_L = (V_L, E_L)$, upper-layer topology $G_U = (V_U, E_U)$, contracted topology $G_C = G_U$, upper-layer edge mapping $M(s) = 0, s \in E_U$, demands on upper-layer edges $D_s, s \in E_U$, capacities on lower-layer edges $C(i) = 0, i \in E_L$.

**Output:** $M(s), C(i)$

1. Pick a cycle $C \in G_C$, find $S_C \subseteq C$ where $s \in S_C$ has edge-disjoint mapping $M(s)$ in $G_L$.
2. Add an edge $t'$ parallel to $t \in (C \setminus S_C)$ and find edge-disjoint mapping for the $t'$ and $t$ in $E_L$. Update $M(t)$.
4. If $G_C$ is not a single node then
5. Go to 1.
6. End if
7. Based on $M(s), s \in E_U$, push $D_s$ flow to the corresponding lower-layer edges used by $M(s)$. Update $C(i), i \in E_L$.

### Algorithm 2 Heuristic for minimizing the spare capacity

**Input:** Upper-layer topology $G_L = (V_L, E_L)$, upper-layer topology $G_U = (V_U, E_U)$, upper-layer edge mapping $M(e), e \in E_U$, demands on upper-layer edges $D_e, e \in E_U$, capacity on lower-layer edges $C(i), i \in E_L$.

1. For all $i \in E_L$ do
2. Find $R_i \subseteq E_U$ where $e \in R_i$ gets disconnected due to the failure of $i$.
3. End for
4. For all $R_i, i \in E_L$ do
5. Find alternative routing with largest residual capacity for $e \in R_i$ under the failure of $i$.
6. End for
7. Calculate all disrupted demands and satisfied demands utilizing alternative routing.

Subproblem 2: In this problem we evaluate the effect of the failure on each lower-layer edge by taking down lower-layer link $i \in E_L$ (representing the link failure) one at a time. The set of disrupted upper-layer edges due to the failure of $i$ (denoted as $R_i$) is recorded. We then calculate the residual capacity available on each lower-layer link after the failure of $i$. For each upper-layer link in $R_i$ an alternative routing which has the largest residual capacity and avoids $i$ in $G_L$ is selected. The demand on a link in $R_i$ is marked as fully satisfied if it can be fulfilled by the alternative routing. Otherwise the demand is labeled as partially satisfied or unsatisfied and the lower-layer edge capacity on alternative routing is assigned to the partial demand which can be satisfied.
4. Preliminary Computational Results
We conduct our simulations utilizing LEMON library [1]. Our experimental lower-layer networks are NSF, EURO 1, EURO 2, G3, and G6 as shown in [2] and illustrated in Fig. 2. The upper-layer network corresponding to each lower-layer network, which is at least two-edge connected, is randomly generated. The initial demands and capacities are generated with the range of 0 to 100. Let the ratio of satisfied and partially satisfied demands to all disrupted demands be the Demand Satisfaction Index. The heuristics were applied 1000 times to the generated lower-layer and upper-layer networks. Based on the second heuristic algorithm, the extra spare capacity required to satisfy all demands after any lower-layer link failure is also calculated and reported in Table 1.

5. Conclusions
In this paper, we present the upper-layer protection for upper-layer survivability in a layered network, which integrates the lightpath routing and lower-layer capacity assignment. We investigate the necessary and sufficient condition for upper-layer survivability under upper-layer protection and provide exact problem formulation. A family of Benders’ cuts and heuristics are generated to solve this problem.

References


Figure 2: Selected lower-layer networks